Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 82731

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

First Semester

Aeronautical Engineering

MA 1101 – MATHEMATICS – I

(Common to ALL Branches)

(Regulations 2004)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If the sum of two eigen values and the trace of a 3×3 matrix A are equal, find the value of $\det(A)$.

2. State Cayley-Hamilton theorem.

3. Find the direction cosines of any straight line segment perpendicular to xy–plane and also to yz–plane.

4. Find the angle between the planes $2x - y + z + 8 = 0$ and $x + y + 2z - 12 = 0$.

5. What is the radius of curvature of the circle $3x^2 + 3y^2 + 6x + 6y - 4 = 0$?

6. Show that the envelope of the family of lines $y = mx + \frac{a}{m}$, m being the parameter is a parabola.

7. If $u = x^2, v = y^2$, find $\frac{\partial(u, v)}{\partial(x, y)}$.

8. If $z = yf(x^2 - y^2)$, show that $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = \frac{xz}{y}$.

9. Convert the equation $x\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + \frac{y}{x} = x^2$ into a linear differential equation with constant coefficients.

10. Find the particular integral of the differential equation $y'' - 3y' + 2y = 2^x$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 5 & 3 \\ 1 & 3 \end{pmatrix}$. Express A^{-1}, if exists in terms of A and I.

(ii) Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$.

Or

(b) (i) Find the value of k for which the equations $x + y + z = 1$, $x + 2y + 3z = k$, $x + 5y + 9z = k^2$ have a solution. (8)

(ii) Let A be a symmetric matrix of order 3 with eigen values $-1, 1, 4$ and corresponding eigenvectors $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ respectively. Find the matrix A. (8)

12. (a) (i) Find the equations of the spheres passing through the circle $x^2 + y^2 + z^2 - 6x - 2z + 5 = 0$, $y = 0$ and touching the plane $3y + 4z + 5 = 0$. (8)

(ii) Find the reflection (image) of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$. (8)

Or

(b) (i) Show that the lines $\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2}$ and $3x - 2y + z + 5 = 0 = 2x + 3y + 4z - 4$ are coplanar. Find their point of intersection and the plane in which they lie. (8)

(ii) Find the angle between the lines whose direction cosines are given by $3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$. (8)

13. (a) (i) Show that the radius of curvature of the curve $r^n = a^n \cosh\theta$ is $\frac{a^n r^{-n+1}}{n+1}$. (8)

(ii) Find the envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are connected by the relation $a + b = c$. (8)

Or

(b) (i) Find the equation of the circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$. (8)

(ii) Find the evolute of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. (8)

14. (a) (i) Find the local minima and maxima of the function of two variables $f(x, y)=x^4+y^4-2x^2-2y^2+2xy$. (8)

(ii) Differentiating $\int_0^x \frac{dx}{x^2+a^2}=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$, evaluate $\int_0^x \frac{dx}{\left(x^2+a^2\right)^2}$ and $\int_0^\infty \frac{dx}{\left(x^2+a^2\right)^2}$. (8)

Or

(b) (i) Find the Taylor's series expansion of $e^x \sin y$ near the point $\left(-1, \pi/4\right)$ upto the third degree terms. (8)

(ii) If $u=u(y-z, z-x, x-y)$, find $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$. (8)

15. (a) (i) Solve $\left(D^2-4D+13\right)y=e^{2x}\cos 3x$. (8)

(ii) Solve $(2x+3)^2\frac{d^2y}{dx^2}-2(2x+3)\frac{dy}{dx}-12y=6x$. (8)

Or

(b) (i) Solve $\frac{dx}{dt}+y=\sin t,\ \ x+\frac{dy}{dt}=\cos t$, given that $x=2$ and $y=0$ at $t=0$. (8)

(ii) Solve, by the method of variation of parameters, $\frac{d^2y}{dx^2}+4y=\sec 2x$. (8)